ISSN 2278-8808 SJIF 2014 = 4.889
An International Peer Reviewed & Referred
SCHOLARLY RESEARCH JOURNAL FOR INTERDISCIPLINARY STUDIES

IMPERFECT REPAIR MODELS INCORPORATING THE COVARIATES: AN APPLICATION

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Abstract

The failure process of repairable systems are generally modeled by using the point processes like renewal process, Nonhomogeneous Poisson process (NHPP), log-linear process etc. using the available failure data. Additional information in the form of covariates can also be utilized in modeling the failure process using imperfect repair models for carrying out more reliable inferences about the process. In this study, we are proposing various imperfect repair models using minimal repair baseline intensity like homogeneous Poisson process (HPP), NHPP, Kijima Type – I and Kijima Type – II process, Arithmetic Reduction of Age process (ARA) and Arithmetic Reduction of Intensity process (ARI), for modeling system failure times. We are also incorporating some covariates which have possible influence on the system failures. We are addressing the problem of point and confidence intervals for the model parameters.

***Keywords**: repairable systems, imperfect repair models, Nonhomogeneous Poisson process, covariates*

1. Introduction

A complex system consists of any structure of more than one component, in which each component performs a particular function. Some examples: industrial or domestic machinery, biological or ecological structures etc. Such systems often observe failures, following which they can be discarded or repaired. For repairable systems, repair action is carried out after a failure occurs which intends to put the system back to the state in which it can perform its function again. Modeling the effect of these repair actions gives valuable insights in order to assess the maintenance and replacement strategies.

The basic assumptions on repair efficiency are: minimal repair (bad – as – old) and perfect repair or maximal repair (good – as – new). But, in reality, the system is in the state between these two extremes: maintenance or repair action reduces the failure intensity but does not bring the system in the state as good as new.

Repairable systems are generally modeled by stochastic processes called point processes. A renewal process is used for systems which undergo major repairs, and after repair they can be termed as good as new system. Here, the failure intensity starts anew from zero and hence the process represents maximal repair.

A non homogeneous Poisson process is used for systems which undergo minor repairs, and the system is brought back to the same state as it was just before failure. Here, the system failure intensity remains the same after repair as it was before repair and hence the process represents minimal repair. Both these processes represents extreme situations, and do not reflect the general behavior of the processes wherein the system is partially repaired and renewed. Such processes can be modeled with imperfect repair models.

Imperfect repair models with renewal process as a baseline have been developed by Kijima (1989), termed as Kijima Type I and Kijima Type II processes. Imperfect repair models with NHPP as a baseline have been developed by Doyen and Gaudoin (2004), termed as Arithmetic Reduction of Age (ARA) and Arithmetic Reduction of Intensity (ARI) processes.

The failure times of a system may be dependent on many factors. Considering some of these factors as covariates, their influence is modeled using appropriate regression model, such as proportional intensity process.

Baseline intensity can be renewal or non homogeneous point process.

We can assume that, the covariates act multiplicatively on the system's failure intensity using a suitable link function, for example, exponential (Cox (1972), Love and Guo (1991), Kumar (1995), Guo et. al. (2001) etc.).

Typical repairable systems include industrial and domestic machinery, but they can also include biological and ecological structures, or can be used for applications arising in social science and commerce. The effect of covariates is to modify the baseline intensity function up or down depending on the nature and strength of the covariates.

2. Modeling the System

Let H_{t^-} represents the history of the process just prior to time t.

The conditional intensity of the process $\lambda(t \mid H_{t^-})$ conditional on the past history of the counting process provides a general framework for the modeling of the point process.

The influence of the covariates can be modeled by means of a proportional intensity process based on Cox (1972), with a maximal repair baseline intensity, and is given by

$$\lambda(t \mid H_{t^-}) = \lambda\left(t - T_{N_{t^-}}\right) f(\gamma' z)$$

A model with minimal repair for the baseline intensity is given by

$$\lambda(t \mid H_{t^-}) = \lambda(t) f(\gamma' z)$$

The link function $f(\cdot)$ can have an exponential, log or logistic form. We are using exponential form as it is highly mathematically tractable.

Thus the baseline intensity with maximal repair will be of the form

$$\lambda(t \mid H_{t^-}) = \lambda\left(t - T_{N_{t^-}}\right) exp(\gamma' z)$$

and that with minimal repair will be of the form

$$\lambda(t \mid H_{t^-}) = \lambda(t) \, exp(\gamma' z)$$

Using Power Law Intensity as a baseline intensity, this can be expressed as

$$\lambda(t \mid H_{t^-}) = \alpha \, \beta \, t^{\beta - 1} \, exp(\gamma' z)$$

For a repairable system, the failure intensity at a point of time depends not only on the mode of operation but also on the history of repairs. Kijima (1989) proposed that the state of the system just after repair can be described by its virtual age, which is smaller than the real age. In this framework, the failure rate depends on the virtual age of the system. Kijima (1989) suggested two repair-effect models. Model 1 assumes that repairs serve only to remove the damage created in the last failure or breakdown (referred to as KT1 Virtual Age Process) whereas Model 2 assumes that repairs could remove all damage accumulated up to that point of time (referred to as KT2 Virtual Age Process). That is, such repairs reset the virtual age of the system to somewhere between that of a good as new system and a minimally repaired system.

The imperfect repair proportional intensity model proposed by Guo and Love (1992) and Gasmi et. al. (2003) is given by

$$\lambda(t \mid H_{t^-}) = \lambda\left(x + \nu_{N_{t^-}}\right) exp(\gamma' z)$$

where ν_t represents the virtual age of the system at time *t*.

Considering ρ as a constant age reduction factor, Guo and Love (1992) used a Kijima Type 1 (KT1) process with the following intensity function:

$$\lambda(t \mid H_{t^-}) = \lambda\left(x + \rho \, T_{N_{t^-}}\right) exp(\gamma' z)$$

Using Weibull density for the time to first failure, they arrived at the model:

$$\lambda(t \mid H_{t^-}) = \alpha \, \beta \left(x + \rho \, T_{N_{t^-}}\right)^{\beta - 1} exp(\gamma' z)$$

Gasmi et. al. (2003) used a Kijima Type 2 (KT2) process, considering ρ as a constant age reduction factor for obtaining the virtual age, and their model is given by:

$$\lambda(t \mid H_{t^-}) = \lambda\left(x + \sum_{j=1}^{N_{t^-}} \rho^{N_{t^-}+1-j} x_j\right) exp(\gamma' z)$$

With Weibull density for the time to first failure, their model is given by:

$$\lambda(t \mid H_{t^-}) = \alpha\,\beta \left(x + \sum_{j=1}^{N_{t^-}} \rho^{N_{t^-}+1-j} x_j \right)^{\beta-1} exp(\gamma' z)$$

Doyen and Gaudoin (2004) proposed Arithmetic Reduction of Age (ARA) and Arithmetic Reduction of Intensity (ARI) with memory '*m*' for characterizing imperfect repair processes. '*Memory*' is a kind of markovian property; it is the maximal number of previous failure times that are involved in the failure intensity.

The ARA model with memory '*m*' (ARA_m) is defined by the failure intensity

$$\lambda(t \mid H_{t^-}) = \lambda\left(t - \rho \sum_{j=0}^{Min\,(m-1,\ N_t - 1)} (1-\rho)^j\ T_{N_t - j} \right)$$

In reduction of age models, it is considered that the repair rejuvenates the system such that its intensity at time *t* is equal to the initial intensity at time A_t , where $A_t < t$.

Thus, the reduction of age models assumes that repair reduces the virtual age of the system by an amount proportional to its age just before the repair.

The ARA_1 proportional intensity process with a power law process as the baseline intensity is given by:

$$\lambda(t \mid H_{t^-}) = \alpha\,\beta\,\left(t - \rho\, T_{N_{t^-}}\right)^{\beta-1} exp(\gamma' z)$$

The reduction of intensity model assumes that repair reduces the failure intensity by an amount proportional to the current failure intensity. That is, the repair action reduces the failure intensity of the system. We are considering only arithmetic reduction, but other types of reduction, like geometric reduction can also be considered.

The ARI model with memory '*m*' (ARI_m) is defined by the failure intensity:

$$\lambda(t \mid H_{t^-}) = \lambda(t) - \rho \sum_{j=0}^{Min\,(m-1,\ N_t - 1)} (1-\rho)^j\ \lambda\left(T_{N_t - j}\right)$$

The ARI_1 proportional intensity process with a power law process as the baseline intensity is given by:

$$\lambda(t \mid H_{t^-}) = \alpha\,\beta\,\left(t^{\beta-1} - \rho\, T_{N_{t^-}}^{\beta-1}\right) exp(\gamma' z)$$

3. Estimation of Parameters

If $0 < t_1 < t_2 < \cdots < t_n$ denote the first n ordered system failure times from a time-truncated process, then the likelihood function of $0 < t_1 < t_2 < \cdots < t_n$ can be given by

$$L(\underline{\theta} \mid data) = \prod_{i=1}^{n} \lambda\,(t_i \mid H_{t^-})\ \exp\left(-\int_0^t \lambda\left(y \mid H_{y^-}\right)\right) dy$$

The most widely used method of estimating the parameters of a process from the repairable systems data is the method of maximum likelihood. Solving the log likelihood equation using iterative methods, the MLEs of the parameters can be obtained.

Using these estimates, we can obtain the local Fisher's information matrix. The inverse of the Fisher's information matrix can be used to obtain the approximate confidence intervals for the parameters.

As the MLEs are asymptotically normally distributed, we can use the asymptotic lognormal distribution for the MLEs and obtain the $100\,(1-\alpha)\%$ confidence interval for the parameter, say τ, as

$$\hat{\tau}\,.\,exp\left(\pm Z_{\frac{\alpha}{2}}\ \sqrt{var\,(\hat{\tau})}\ /\ \hat{\tau}\right)$$

where $\hat{\tau}$ is the MLE of parameter τ, $Z_{\frac{\alpha}{2}}$ is the upper $\frac{\alpha}{2}$ percentile point of a standard normal distribution and $var\,(\hat{\tau})$ is the appropriate diagonal entry corresponding to the parameter τ of the inverse of Fisher's Information matrix.

Also, the Akaike's Information Criterion (AIC) and Bayesian Information Criterion (BIC) can be used to choose the best model for modeling the system failure times. These criterions are defined as

$AIC = -2\max\,\ell + 2\,m$ and $BIC = -2\max\,\ell + m\,ln\,n$

where, *m* is the number of estimated parameters, *n* is the number of observations and max ℓ is the maximized log – likelihood.

4. An Application

Many researchers are interested in studying stock market efficiency across the globe. Mishra (2009) examined the efficiency of Indian stock market using Random Walk and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models. The Power Law Process has been used in previous studies to analyze duration dependence in economic and financial cycles. Zhou and Rigdon (2008, 2011) analyzed duration dependence in U.S. Business cycles analysis using the Modulated Power Law Process (MPLP).

Shah and Muralidharan (2013) have analyzed Indian stock prices using non homogeneous Poisson process models like Power Law process and Log Linear Process. It is known that the variation in SENSEX is influenced by many factors, like, Crude prices, CRR, Food Price Inflation, FDI, Dollar Exchange Rate, FOREX etc. In previous studies (Gulati and Kakhani (2012), Ghosh, Roy, Bandopadhyay and Choudhuri), it has been shown that the Price of USD Vs. INR, Oil Prices, CRR etc. have significant impact on SENSEX.

In this study, we are using imperfect repair proportional intensity models with non homogeneous baseline intensity process, to model the fluctuations in the BSE SENSEX during January 2009 to November 2013, along with the price of USD in terms of INR as a covariate, which is one of the influencing factors for BSE SENSEX index.

The monthly high BSE SENSEX figures are considered as the variable of interest. The event times are considered as the number of months it took to observe a fluctuation of 500 points in BSE SENSEX, which leads to their modeling using point processes.

The fluctuation of 500 points in BSE SENSEX is taken as an event time and correspondingly, a marginal percentage change in USD vs. INR exchange rate is assumed to take value $Z = 1$, otherwise $Z = 0$.

Thus, we have the following data:

t = cumulative failure times / event times (here, the number of months after which the fluctuation

of 500 points is observed in BSE SENSEX)

The cumulative failure times are as follows:

1, 2, 3, 4, 5, 8, 12, 13, 14, 16, 18, 20, 21, 25, 26, 29, 32, 33, 35, 37, 39, 44, 47, 48, 51, 52, 53, 55, 56, 58

The corresponding values of covariate Z are as follows:

1, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 1, 1, 0, 1, 1, 1, 0, 0, 0, 1, 0, 1, 1, 1

We have in all n = 30 event times during the period January 2009 to November 2013.

Table 1: Parameter Estimation for HPP with Covariate Z

Parameter	Point Estimate	95% Confidence Interval
λ	1.2499	(0.9018, 1.7395)
γ	-19.4847	(-12.3491, 6.2482)

Other Values:

$\max \ell$ = -23.30569, AIC = 50.61138, BIC = 53.41377

Table 2: Parameter Estimation for PLP with Covariate Z

Parameter	Point Estimate	95% Confidence Interval
α	0.5963	(0.3726, 0.8425)
β	0.9384	(0.5928, 2.2496)
γ	0.1758	(0.09736, 0.3945)
Other Values:		
$\max \ell$ = -49.5755,	AIC = 105.1511,	BIC = 109.3547

Table 3: Parameter Estimation for KT1 Process with Covariate Z

Parameter	Point Estimate	95% Confidence Interval
α	6.1417	(3.8479, 14.3547)
β	1.2385	(0.7149, 2.6538)
ρ	0.0145	(0.0097, 0.05233)
γ	1.3673	(0.9767, 3.8090)
Other Values:		
$\max \ell$ = -49.5755,	AIC = 107.151,	BIC = 112.7558

Table 4: Parameter Estimation for KT2 Process with Covariate Z

Parameter	Point Estimate	95% Confidence Interval
α	9.6467	(5.4467, 22.8974)
β	2.2385	(1.0735, 4.357)
ρ	0.0065	(0.0045, 0.0335)
γ	1.9167	(1.1072, 5.3506)
Other Values:		
$\max \ell$ = -67.5636,	AIC = 109.4899,	BIC = 122.3495

Table 5: Parameter Estimation for ARA_1 Process with Covariate Z

Parameter	Point Estimate	95% Confidence Interval
α	6.8272	(3.5548, 15.3472)
β	2.2184	(1.4789, 5.3629)
ρ	0.1296	(0.0836, 0.4596)
γ	1.6361	(0.9261, 3.3861)
Other Values:		
$\max \ell$ = -50.5755,	AIC = 103.5398,	BIC = 108.2903

Table 6: Parameter Estimation for ARI_1 Process with Covariate Z

Parameter	Point Estimate	95% Confidence Interval
α	7.1025	(4.9269, 15.7611)
β	2.0284	(1.4684, 5.0584)
ρ	0.1309	(.0969, 0.3349)
γ	1.6938	(1.1338, 4.4938)
Other Values:		
$\max \ell$ = -51.0573,	AIC = 110.1146,	BIC = 115.7194

Concluding Remarks:

It can be seen that the $KT1$ process with a power law process as the baseline intensity and covariate USD vs. INR exchange rate provides a better fit to the data set as the AIC and BIC values are minimum corresponding to this process.

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